Characterizing potentials using the structure of a one-dimensional chain demonstrated using a dusty plasma crystal

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A procedure was developed to characterize the interparticle potential in a lattice that is confined by an external potential. The first of the two steps is to characterize the confining potential, which can be done using various schemes involving observations of particle motion. The second step is to characterize the interparticle potential using measurements of the equilibrium particle positions. This can be done with either of two methods developed here, a force-balance method or a simpler equation-of-state method. To demonstrate and test these methods, an experiment and a molecular dynamics simulation were performed with a one-dimensional Coulomb chain of particles confined in a parabolic potential. The experiment used a dusty plasma consisting of charged microspheres levitated in the plasma sheath above a narrow groove in a lower electrode.

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I. INTRODUCTION

We develop a procedure to characterize the interparticle potential in a lattice that is confined by an external potential, based on knowledge of the external potential and equilibrium positions of the particles. This procedure is useful for all kinds of interparticle potential, but here we will apply it only to the case of mutually repulsive particles.

Charged particles of the same polarity are mutually repulsive, and when they are confined in an external potential, they can arrange themselves into a lattice structure. Electrons at the surface of liquid helium can form into a two-dimensional Wigner crystal [1,2], or they can be confined into a one-dimensional charged system [3]. Atomic ions can form into a one-dimensional Coulomb chain in a linear Paul trap [4], or a two-dimensional lattice in a cylindrical Penning trap [5].

The structure of a lattice subjected to an external potential is determined by the balance between the interparticle potential and the external potential. The mutual repulsion between like charges causes an outward pressure, while the external potential counteracts this repulsion, forcing the particles together. Adding more particles compresses a crystal, reducing the particle spacing.

In a dusty plasma, small particles of solid matter are electrically charged and suspended in a plasma. Under some conditions, when there is sufficient damping, the particles will arrange themselves into a lattice structure termed plasma crystal. A plasma crystal can be a three-dimensional suspension [6], a two-dimensional lattice [7–9], or a one-dimensional chain [10,30]. Like a conventional molecular solid, this kind of crystal exhibits solid behavior such as a melting-freezing transition [11,12] and phonon propagation [13–15].

In a plasma crystal, the external potential is provided by a sheath [16] above a horizontal lower electrode. In the vertical direction, the sheath has a strong electric field that levitates particles against the downward force of gravity. In the horizontal direction, particles are confined due to the curvature of the sheath edge, corresponding to a small inward horizontal component of the sheath electric force, which compresses the particles together.

In a plasma crystal, the interaction between charged particles is a Coulomb repulsion, which is screened by the ambient plasma. For only two particles levitated on the same horizontal plane in a sheath, Konopka *et al.* [17] demonstrated that the binary interaction was accurately modeled by a Yukawa potential,

$$\phi(r)=\frac{Q^2}{4\pi\epsilon_0}\frac{e^{-r/\lambda_D}}{r}, \tag{1}$$

where r is the distance between two particles, Q is the particle charge, and λ_D is the shielding length. In that experiment, only the potential in the horizontal plane were characterized.

The potential can, however, be anisotropic with an small attractive element, due to an ion wakefield downstream of a particle in a flowing plasma [18]. This wakefield has a significant effect for a multilayer particle suspension in a sheath. Nevertheless, for a monolayer suspension, the wakefield has little effect in the horizontal plane where the particles are suspended. Thus, it is common to model the interaction for a monolayer plasma crystal as a simple Yukawa potential [19], as we shall do here.

For a dusty plasma, various methods have been developed for measuring Q and λ_D, and these methods can be grouped according to whether they rely on measurements of particle motion or equilibrium particle positions. Methods relying on particle motion include vertical resonance vibration [20], particle mean-square displacement [21], and dispersion-relation fitting [22,23]. Methods using measurements of equilibrium particle position include approaches relying on measurements of particle spacing within a two-dimensional lattice [24] and of height in the vertical electric field of the sheath [16].

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We have developed a complete procedure to characterize the interparticle potential. As presented here, the procedure works for a one-dimensional chain, i.e., a single row of particles of limited length. It might also be possible to extend the procedure to two or three dimensions. This procedure has two steps. First, the confining potential is determined. Second, the interparticle potential is characterized using either the force-balance method of Sec. III or the equation-of-state method of Sec. VIII. Both of these methods require measuring equilibrium particle positions, and they assume that the confining potential is the same, regardless of the length of the chain. These two methods require measurements for chains of at least two different lengths, if the interparticle potential has two free parameters, e.g., Q and λ_D for a Yukawa potential

We apply this procedure to a plasma crystal. We carry out both steps of the procedure, characterizing the profile of the confining potential and then the interparticle potential between charged particles suspended in a plasma. Another application of the force-balance method is as a sensitive sheath diagnostic tool, as we will discuss.

II. CHARACTERIZING THE CONFINING POTENTIAL

The first step of the procedure is to determine the profile of the confining potential. How this is done will depend on the particular physical system. Here we review two methods that have been used for a plasma crystal: single particle motion and center-of-mass oscillation.

For a single particle in the plasma sheath, the profile of the confining potential can be derived by analyzing the trajectory of the particle motion in the horizontal plane. Konopka *et al.* [17] used a positively-biased probe to displace a single particle and then observed its motion as it was restored toward its equilibrium position. Another approach, which works even if the confining potential profile is not parabolic, is to accelerate a single particle using laser radiation pressure [25]. From the particle's trajectory, one can calculate the confining potential, as well as radiation pressure and gas drag.

The confining potential can be also characterized by the frequency of center-of-mass oscillation, which is the same for multiple particles as for a single particle. One can observe the motion of all particles in a one-dimensional chain and calculate the velocity of the center of mass of the particles. From the spectrum of this velocity, one can obtain a resonance frequency, ω, Ref. [26], which is sometimes termed the sloshing-mode frequency. The confining potential can then be calculated from ω, if the confining potential has a parabolic shape. To verify that the profile of the confining potential has a parabolic shape, one can check that harmonics of ω are absent from the spectrum.

All of the methods described above to characterize the confining potential rely on measuring the motion of particles, and therefore require knowledge of particle mass. After characterizing the confining potential, the next step is to use either the force-balance method of Sec. III or the equation-of-state method of Sec. VIII to characterize the interparticle potential. These two methods are based on measurements of equilibrium positions of particles, and therefore do not require any knowledge of particle mass.

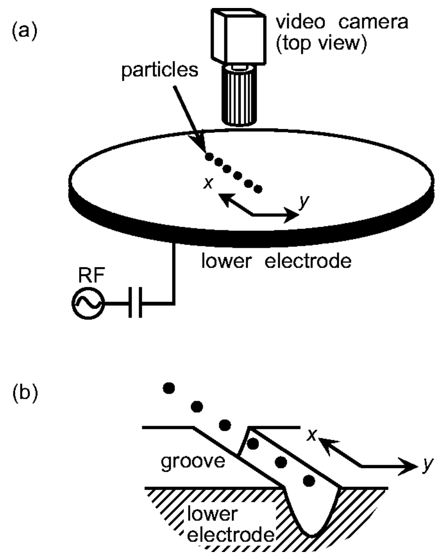

FIG. 1. (a) Sketch of the apparatus. (b) Sketch of particles, levitated in a plasma sheath above a groove in the lower electrode. The combination of an interparticle repulsive potential and the confinement, provided by an electric sheath that conforms to the shape of the electrode, causes the particles to arrange themselves into a one-dimensional chain. Particles are imaged using a video camera from above.

The confining potential in our experiment is provided by the apparatus shown in Figs. 1 and 2. Particles are levitated in a sheath, and this sheath has a curvature so that particles are confined by a bowl-shaped potential that is parabolic in every direction. The combination of a repulsive interparticle potential and the confining potential causes the particles to arrange in a one-dimensional chain, as shown in Fig. 3. More details of the experiment are presented in Sec. VI.

III. FORCE-BALANCE METHOD

The second step of the procedure is to use either the force-balance method presented here, or the simpler equation-of-state method in Sec. VIII, to characterize the in-

FIG. 2. Photograph of the lower electrode. Particles are shaken into the plasma; they settle into the sheath above the electrode. Due to the curvature of the sheath, they collect, forming a one-dimensional chain above the central groove.

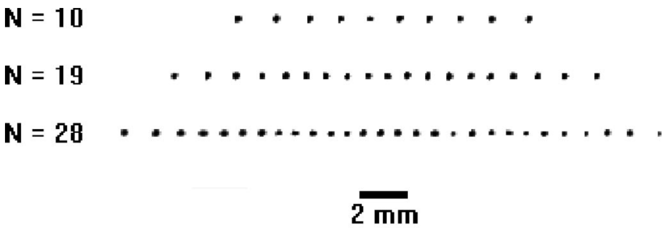

FIG. 3. Image of chains of three different lengths. The particle spacing a decreases with N. Within a chain, the particle spacing is compressed more in the center, due to the confining potential. The three images shown here were recorded separately; there was only one chain presented above the groove at a time.

terparticle potential. The development of these two methods is the chief purpose of this paper.

In this section, we develop the force-balance method for a lattice that is confined by an external potential. This method can be used to characterize the interparticle potential using knowledge of the external confining potential, as we shall do in this paper. (Alternatively, if the interparticle potential and its parameters were known, this method could be used in reverse to determine the confining potential.)

The force-balance method is based upon a zero net force acting on each particle i in a lattice. Including the repulsions applied by other particles and the external confining potential, the net force is zero if the particle i is at its equilibrium position,

$$\sum_j \nabla\phi(\mathbf{r}_i,\mathbf{r}_j)+Q\mathbf{E}(\mathbf{r}_i)=\mathbf{0}, \quad i=1,2,\ldots,N, \tag{2}$$

where ϕ is the interparticle potential for the particles at positions $\mathbf{r}_i$ and $\mathbf{r}_j$, and $\mathbf{E}(r_i)$ is the external electric field at $\mathbf{r}_i$. In Eq. (2) we also assume the particles are identical, with the same charge Q.

Equation (2) is used to find the interparticle potential ϕ, and the way this is done will depend on the number of free parameters in the interparticle potential. If the interparticle potential has only one parameter, for example Q in the case of a bare Coulomb repulsion $Q^2/4\pi\epsilon_0|\mathbf{r}_i-\mathbf{r}_j|$, then it is sufficient to apply Eq. (2) to a single snapshot showing the equilibrium positions of the particles in a single lattice. If the potential has two parameters, for example, Q and λ_D in the case of a Yukawa repulsion, then Eq. (2) must be applied using particle positions for not just one, but two different lattices. These two lattices can be distinguished by a different number of particles N.

Using Eq. (2), the force-balance method can be implemented as follows: (i) measure particle positions in a lattice that is confined by an external potentials; (ii) establish the relations between interparticle potential and the external confining potential from Eq. (2); (iii) if the interparticle potential has more than one parameters, find these parameters using a graph, in which the relation for each lattice is represented by a curve and the intersection of the curves yields the desired parameters; (iv) confirm that the particle positions and the result for the interparticle potential are consistent with the form of the confining potential that we used. Finally, if the confining potential is not parabolic, it may be necessary to perform an iterative loop to refine the parameters for the confining potential and the interparticle potential.

In comparison to other methods that require measurements of particle motion, the force-balance method requires only a few snapshots of particle positions, and it does not need information of local plasma conditions in the sheath. If one chooses the simpler equation-of-state method in Sec. VIII, our procedure is probably quicker than most methods relying on measurements of particle motion.

Our force-balance method is comparable to an earlier method that was reported for a two-dimensional plasma crystal [24]. Both are based upon a balance of force or pressure in a lattice, and they both use measurements of particle positions. Our force-balance method, however, differs from the method of Ref. [24] in two ways. First, we use the exact position of each particle rather than discarding this information by assuming a continuous medium. Second, we find the relations between the interparticle potential and the external confining potential, corresponding to lattices of two or more sizes, and then we find the parameters they have in common. In contrast, in Ref. [24] the particle spacing or crystal size was fit to a function of particle number.

IV. FORCE-BALANCE METHOD FOR A YUKAWA INTERACTION

In this section, we develop the force-balance method specifically for a plasma crystal, with a Coulomb interaction that is screened by an ambient plasma. Here, we assume that the interparticle potential is modeled as a Yukawa repulsion, and the confining potential is provided by a curved sheath. We neglect any additional forces a particle might experience in a plasma [27,28], and we assume that the only significant forces in determining lattice structure in the horizontal direction are the particle-particle interaction and the sheath electric force. This is a reasonable assumption for one- and two-dimensional plasma crystals because the other forces, such as gravity, usually act in the direction perpendicular to a monolayer suspension.

In a plasma crystal, the force balance, Eq. (2), becomes

$$QE(\xi_i)=\frac{Q^2}{4\pi\epsilon_0\lambda_D^2}\sum_{j\neq i}\frac{\xi_j-\xi_i}{|\xi_j-\xi_i|^3}(1+|\xi_j-\xi_i|)e^{-|\xi_j-\xi_i|}, \tag{3}$$

where $\xi_i=x_i/\lambda_D$, x_i is the particle's position, and $E(\xi_i)$ is the electric field due to the confining potential. Here, we assume that all the particles are on a common axis, i.e., they form a one-dimensional chain. It might also be possible to develop Eq. (2) for a two or three-dimensional lattice, but the math would be less simple.

Equation (3) is written in a form that can be used for a confining potential of arbitrary shape. Hereafter, however, we will assume a parabolic confining potential. This assumption can be verified experimentally as described in Sec. II. In fact, most plasmas do have a potential profile with a maximum that can be locally approximated by a parabola. In case that the potential is not parabolic, our method could in principle be extended by performing an iterative loop.

For a parabolic confining potential $QE(\xi)=m\omega^2\xi$. Thus, Eq. (3) reduces to

$$\eta\xi_i=\frac{1}{2\lambda_D^3}\sum_{j\neq i}\frac{\xi_j-\xi_i}{|\xi_j-\xi_i|^3}(1+|\xi_j-\xi_i|)e^{-|\xi_j-\xi_i|} \qquad (4)$$

for the ith particle at position ξ_i. Here we have defined a variable

$$\eta=\frac{2\pi\epsilon_0 m\omega^2}{Q^2}, \qquad (5)$$

which has units of m^{-3} and serves as a measure of the relative importance between the Coulomb repulsion in the denominator, and the external confining potential in the numerator. The right side of Eq. (4) only depends on particle positions and λ_D. Using a value for λ_D, we calculate the right side of Eq. (4) for each particle i, and linearly fit the data for $\eta\xi_i$ from Eq. (4) vs ξ_i to a straight line. This yields the slope of the line, which is η. We then repeat for other values of λ_D, yielding a curve for η vs λ_D. We term this curve as the "parameter curve."

The parameters for the interparticle potential are obtained from the intersection of the parameter curves of η vs λ_D for a minimum of two chains, each with a different length. This method of solution assumes that the chains were measured under the same experimental conditions, so that the confining potential and the parameters for the interparticle potential were the same. In a plasma crystal, this requires that the chains should be formed at same discharge conditions.

V. SIMULATION TEST OF THE FORCE-BALANCE METHOD

As a test of the force-balance method, we performed a molecular dynamics simulation. We integrated each particle's equation of motion, $m\ddot{\mathbf{r}}_i=-\nabla(\Sigma\phi_{ij}+\Phi_i^{ext})-\nu_E m\dot{\mathbf{r}}_i$, where ϕ_{ij} is the binary Yukawa interparticle interaction of Eq. (1). The confining potential is parabolic, $\Phi_i^{ext}=m(\omega_x^2x_i^2+\omega_y^2y_i^2)/2$, and the gas drag is $\nu_E m\dot{\mathbf{r}}_i$. Here, $\mathbf{r}_i=(x_i,y_i)$ is measured from the minimum of the confining potential. We allowed the simulation to run until all particle motion was damped and the particles had settled into equilibrium positions. It is important to note that Q and λ_D are input parameters for the simulation.

We used the particle equilibrium positions generated by the simulation as the input for the force-balance method in Sec. IV. We produced parameter curves of η vs λ_D for three chains: $N=16$, 26, and 45, as shown in Fig. 4. We found the three curves intersect not at a common point, but nearly so, with crossings at $\lambda_D=0.61$, 0.66, and 0.71 mm.

Our test is to compare these values produced by the force-balance method with the value that was assumed in the simulation, $\lambda_D=0.66$ mm. We conclude that the result from the force-balance method is close to the actual value, with an accuracy of approximately 8%.

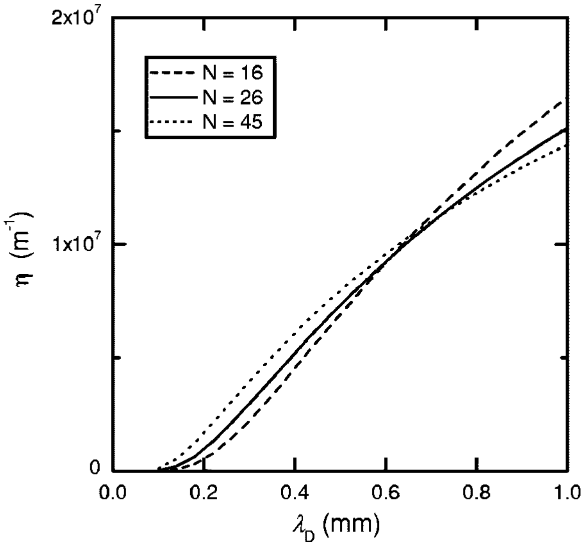

FIG. 4. Simulation results for the force-balance method for a one-dimensional chain with a parabolic confining potential and a Yukawa interparticle potential. We computed η, which is a measure of the relative importance of the confining potential and interparticle potential, from Eq. (4) using the particle positions after the simulation reaches equilibrium. The variation of η with λ_D is a parameter curve. The parameter curves for different chain lengths cross at nearly, but not exactly, the same point. This intersection yields the parameters Q and λ_D for the interparticle potential.

VI. EXPERIMENT

Here we describe an experimental test of the force-balance method, which we will also use to test the equation-of-state method in Sec. VIII. In the experiment, we formed one-dimensional chains of various lengths in a radio frequency (rf) plasma and we measured the particle equilibrium positions.

We used the experimental setup sketched in Fig. 1(a). A plasma was produced in a capacitively-coupled rf discharge, using a 13.56 MHz rf voltage with a peak-to-peak amplitude of 94 V, yielding a self-bias of 48 V. A sheath formed immediately above the lower electrode, shown in Fig. 2. We used xenon gas at a low pressure of 5 mtorr. The plasma had a density of 1.2×10^9 cm^{-3} and an electron temperature of 1.6 eV, as measured by a Langmuir probe in the main plasma, not in the sheath where the particles were levitated. We used a shaker to introduce a small number of particles with a diameter of 8.09 μm, as measured by TEM, and a mass density of 1.514 g/cm^3, as reported by the manufacturer.

To image the particles, we illuminated them with a He-Ne laser sheet and viewed with a video camera at 29.97 frames per second. The camera has a field of view of 13 mm$\times$10 mm. The video signal was digitized by a 8-bit monochrome frame grabber and recorded as a series of images with a resolution of 640$\times$480 pixels. Particle positions were then measured in each frame with subpixel spatial resolution [29]. A particle's velocity was calculated by subtracting its positions in two consecutive frames.

A one-dimensional chain was externally confined by the natural electric fields in the sheath above the lower electrode. The sheath conforms to the shape of the electrode, which had a groove-shaped depression along the x direction, as shown in Fig. 2, to form a one-dimensional chain. Everywhere along the groove's length, it has a parabolic shape in the y direction, as shown schematically in Fig. 1(b), with a depth $z=y^2-4$, where z and y are both measured in mm. This depth was uniform with respect to x, the position along the groove.

The first step of our procedure is to characterize the confining potential along the chain, which can be done in several ways, as described in Sec. II. One way is to manipulate a single particle using lasers. We found that the resonance frequency of a single particle was 0.12 Hz, corresponding to $\omega=0.75\ \mathrm{s}^{-1}$, along the x axis. Another way to characterize the confining potential is to measure the frequency spectrum of particle's natural motion. We did this for three chains of different length, and averaged the three results, yielding $\omega=0.80\ \mathrm{s}^{-1}$, for the x direction. Because this value is an average of three measurements, and because it is not based on the motion of a single particle (a single particle might not happen to have a size in the middle of the particle size dispersion), we believe it is more accurate than the measurement for a single particle.

The confinement in the vertical direction, with a vertical resonance frequency of 15 Hz, was strong enough to prevent any vertical buckling of the lattice. In a test, we verified that, as additional particles were added to the chain there was no change of the vertical resonance frequency, and therefore Q was independent of N.

VII. EXPERIMENTAL RESULTS FOR FORCE-BALANCE METHOD

Figure 3 shows raw images for chains of different lengths, $N=10$, 19, and 28. Adding more particles to a chain causes the chain to be more compressed, i.e., a decreases with N. Computing the average of the individual particle spacing

$$a=(N-1)^{-1}\sum_i (x_i-x_{i-1}) \qquad (6)$$

in Fig. 3, we find $a=1.25$, 0.8, and 0.73 mm, corresponding to $N=10$, 19, and 28, respectively.

Within the chain, the particle spacing is not uniform. The confining potential causes the chain to be compressed at the center. This can be seen in Fig. 3, where a is smallest in the center of the chain, and largest at the ends.

In the force-balance method we find a parameter curve relating the interparticle potential and the confining potential for each chain, and then we find the intersection of the curves for chains of different lengths, yielding the desired parameters for the interparticle potential. For a Yukawa chain with a parabolic confining potential, this is done using Eq. (4) to compute η from the measured particle positions, for a particular value of λ_D. Repeating this calculation for a range of value of λ_D yields the parameter curve. We prepared parameter curves for a minimum of two chain lengths. We then find the intersection of two parameter curves, yielding the parameters for the interparticle potential, which are Q and λ_D in the case of a Yukawa potential. Results are shown in Fig. 5,

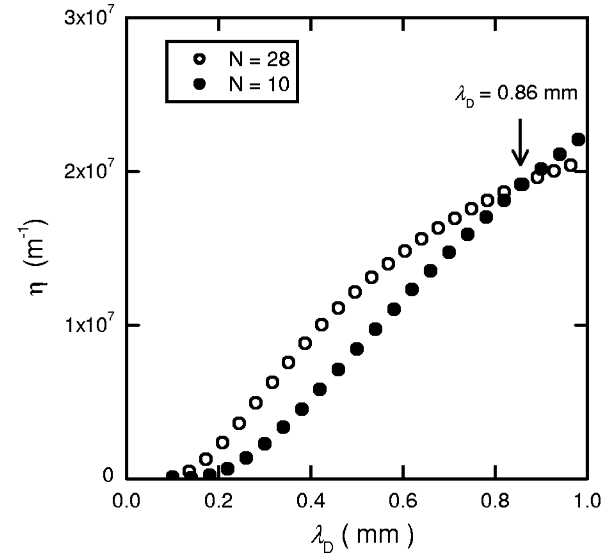

FIG. 5. Experimental results for the force-balance method. Parameter curves of η vs λ_D are shown for $N=10$ and 28. The intersection of these curves yields Q and λ_D, the parameters for the interparticle potential.

for two chain lengths, $N=10$ and 28. The two chains were formed at same plasma conditions, so that they should have the same Q and λ_D.

We now present results for $N=10$ and 28. Figure 5 shows the intersection is at $\lambda_D=0.86$ mm, which is one of the two desired parameters. From the value of η at the point of intersection, we used Eq. (5) to compute $Q/\omega=7800e\ \mathrm{s}^{-1}$, where e is the elementary charge. Using $\omega=0.8\ \mathrm{s}^{-1}$ from Sec. VI yields $Q=6200e$, which is the other desired parameter.

We performed a test to determine how sensitively the result depends on the experimental precision in measuring particle positions. In our test, in the shorter chain which had a length of 600 pixels, we altered the position of a single particle by a displacement of one pixel, which is larger than the subpixel uncertainty in particle position. This resulted in a change of λ_D of 0.1%.

Next, in the force-balance method, we confirm that the confining potential is a parabola. This is done using the results for Q and λ_D from above. Figure 6 shows the electric field, calculated from Eq. (3), as a function of particle position, for $N=10$. The electric field is almost linear, which corresponds to a parabolic potential. We note that calculating the electric field in this way can be used as a sensitive diagnostic tool for the sheath in a plasma. A small number of particles, levitated in a plasma, can be used to measure the potential profile, in the direction parallel to the electrode, in the sheath.

As a test, we now compare our results for Q and λ_D to values obtained using another method. We observed the natural motions of the one-dimensional chains and measured their dispersion relations using a method similar to that of Ref. [23]. Comparing the measured dispersion relations to a theory [30] that assumes an infinite chain length and a uniform a, we found $Q=7800e$ and $\lambda_D=0.88$ mm. These val-

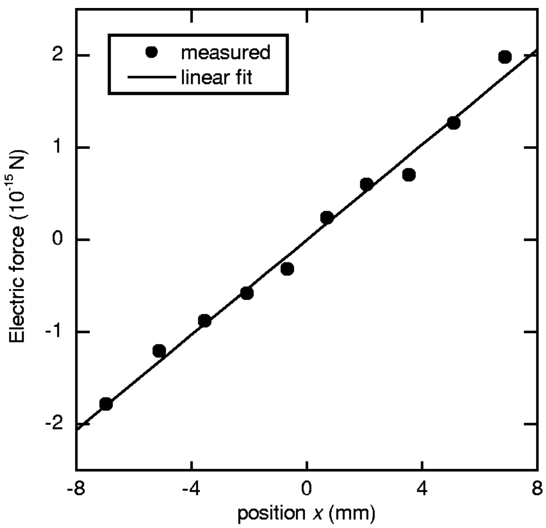

FIG. 6. Profile of the horizontal electric field in the sheath of a plasma. We computed the electric field using the experimental particle positions and Eq. (3). The data fit a straight line; this test serves as a confirmation that the confinement is a parabolic potential.

ues are close to the results of the force-balance method presented above. From this comparison, we conclude that the force-balance method can yield reasonable values of the parameters.

VIII. EQUATION-OF-STATE METHOD

In this section, we present an equation of state, describing the relation between lattice constants, such as a and N, and the parameters for the interparticle potential. This equation of state is developed as an alternative to the force-balance method; either method can be used as the second step of our procedure. This equation-of-state method requires only a single step of solving two equations for two unknown variables, η and λ_D. The solution can be found easily using a graph of η vs λ_D. This graph has a form similar to Fig. 5 for the force-balance method, but the data plotted for η are obtained in a simpler way. As a result, this method can yield an estimation of Q and λ_D much more rapidly than the force-balance method, although it is less accurate due to the approximation that we assume a uniform a.

For any lattice, an equilibrium particle spacing corresponds to a minimum potential energy, i.e., the first derivative of the potential energy U with respect to a is zero. For a one-dimensional chain with a parabolic confining potential and a Yukawa interparticle potential, the total potential energy is

$$U=\frac{m\omega^2a^2}{4}\sum_{i=1}^{N/2}(2i-1)^2+(N-1)\frac{Q^2}{4\pi\epsilon_0}\frac{e^{-a/\lambda_D}}{a}+(N-2)\frac{Q^2}{4\pi\epsilon_0}\frac{e^{-2a/\lambda_D}}{2a}. \tag{7}$$

This is an approximation because it assumes that a is uniform and it includes only the interactions with the four nearest neighbors. Taking the first derivative of Eq. (7) and solving $dU/da=0$ yields an equation of state

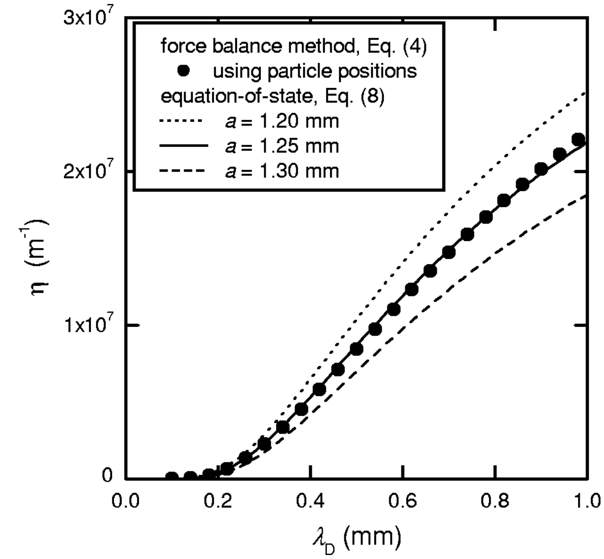

FIG. 7. Test of the equation-of-state method's sensitivity to a. Data are shown for $N=10$. Using $a=1.25$ mm, which is the average particle spacing in the experiment, the equation-of-state method yields the solid line, which agrees well with the data points from the force-balance method. Curves for the equation-of-state method are also shown for the values of a that bracket the average value of 1.25 mm to illustrate the sensitivity of the parameter curve to the value of a.

$$\eta a^3=\frac{N-1}{\sum_{i=1}^{N/2}(2i-1)^2}\left[\frac{1+\kappa}{e^\kappa}+\frac{(N-2)}{(N-1)}\frac{(1+\kappa)}{e^\kappa}\right], \tag{8}$$

where $\kappa=a/\lambda_D$.

The equation of state in Eq. (8) can be used for various purposes, depending on the information that is known. If the average particle spacing a and particle number N are known, it is possible to compute a parameter curve of η vs λ_D, similar to that in the force-balance method, e.g., Fig. 5. One can then find the intersection of a minimum of two parameter curves for chains of different lengths. This is our primary use of Eq. (8). Another use of Eq. (8) is to predict the particle spacing a, if the interparticle potential and the confining potential are known. Although we have not done so here, this might be useful, for example, in planning an experiment.

Using this equation-of-state method for our experimental data, we obtained $\lambda_D=0.82$ mm and $Q=5900e$. These are reasonably close to the values $\lambda_D=0.86$ mm and $Q=6200e$ obtained from the force-balance method. The force-balance method should be more accurate than the equation-of-state method, because it does not ignore the nonuniform particle spacing and the interactions with particles beyond the four nearest neighbors. Nevertheless, the simple method, which requires much less computational effort, yields almost same result.

In using the equation-of-state method, one must first calculate a suitable representative value of the interparticle spacing from the experimental data. Recall that in the experiment, the particle spacing is not uniform, but is compressed near the center of the chain. To choose a single value of a to use in this method, we simply compute the average of the individual spacings using Eq. (6). Using $a=1.25$ mm and Eq. (8), we calculated a parameter curve for $N=10$, as shown in Fig. 7. We note that the agreement is very good with the data points which were computed using Eq. (4) for the force-balance method.

As a test of the sensitivity of the equation-of-state method to the value of a that is used, we also show parameter curves, calculated using Eq. (8), for values of a that bracket the average $a=1.25$ mm. The error in Q and λ_D will depend on not only the uncertainty in a, but also the slope of a parameter curve for a second value of N, which would intersect the curve shown in Fig. 7. If the second curve were from the chain with $N=28$, a ± 0.05 mm uncertainty in a will yield a 30% uncertainty in λ_D.

IX. CONCLUSIONS

A complete procedure was developed to characterize the interparticle potential for a lattice that is confined by an external potential. This procedure includes two steps: characterizing first the confining potential and then the interparticle potential using knowledge of particle positions. This procedure was demonstrated in an experiment using charged particles in a plasma crystal arranged in a one-dimensional chain.

In the first step of the procedure, one must characterize the spatial profile of the confining potential. In the second step, the interparticle potential is characterized using either the force-balance method of Sec. III, or the equation-of-state method of Sec. VIII. One first computes parameter curves for lattices of different sizes, and then from the intersection of the curves one can find the desired parameters for the interparticle potential.

The force-balance method is based upon the balance between mutual repulsion and external confinement. We tested this method using a molecular dynamics simulation. As an alternative to the force-balance method, the equation-of-state method was also developed, based upon an equation of state. The equation of state can predict a parameter curve, if the lattice constants, such as particle spacing and number, are known.

The potential profile in the sheath was also diagnosed using the force-balance method, verifying that the confining potential was parabolic in the experiment.

Finally, note that, after preparing this paper, we learned a method recently reported by Hebner and Riley [31] for determining the interparticle potential for a one-dimensional chain confined by a parabolic potential in a dusty plasma. Their method is comparable to the force-balance methods in Secs. III and IV, beginning with the same equations for force balance, but to determine Q and λ_D they use a fitting method rather than an intersection of parameter curves. The present paper includes a force-balance method similar to that of Ref. [31], as well as an equation-of-state method.

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